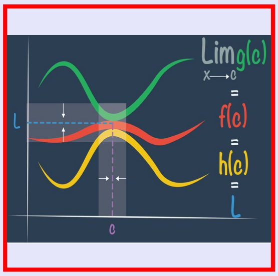
Calculus I

Lecture 10

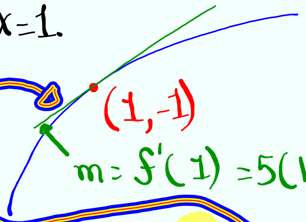


Feb 19-8:47 AM

Class QZ 7

Box Your Final Ans.

Find eqn of the tangent line to the graph of $f(x)=x^5-2\sqrt{x}$ at $x=1$.



$f(x)=x^5-2\sqrt{x}$

$f(1)=1^5-2\sqrt{1}=\boxed{-1}$

$(1,-1)$

$m=f'(1)=5(1)^4-\frac{1}{\sqrt{1}}=5-1=\boxed{4}$

$f(x)=x^5-2x^{\frac{1}{2}}$

$f'(x)=5x^4-\cancel{2}\cdot\frac{1}{\cancel{2}}x^{-1/2}$

$f'(x)=5x^4-\frac{1}{\sqrt{x}}$

$y-(-1)=4(x-1)$

$y+1=4x-4$

$\boxed{y=4x-5}$

Jul 2-7:15 AM

Differentiation formulas:

1) $\frac{d}{dx}[c]=0$ 2) $\frac{d}{dx}[x]=1$

3) $\frac{d}{dx}[x^n]=n\,x^{n-1}$ 4) $\frac{d}{dx}[c\,f(x)]=c\,\frac{d}{dx}[f(x)]$

5) $\frac{d}{dx}[f(x)+g(x)]=\frac{d}{dx}[f(x)]+\frac{d}{dx}[g(x)]$

6) $\frac{d}{dx}[f(x)-g(x)]=\frac{d}{dx}[f(x)]-\frac{d}{dx}[g(x)]$

7) $\frac{d}{dx}[f(x)\,g(x)]=\frac{d}{dx}[f(x)]\,g(x)+f(x)\,\frac{d}{dx}[g(x)]$

8) $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{\frac{d}{dx}[f(x)]\,g(x)-f(x)\cdot\frac{d}{dx}[g(x)]}{[g(x)]^2}$

Jul 2-8:19 AM

Prove $\frac{d}{dx}[c\,f(x)]=c\,\frac{d}{dx}[f(x)]$

Recall $\frac{d}{dx}[f(x)]=f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$

$$\frac{d}{dx}[c\,f(x)]=\lim_{h\to0}\frac{\overset{\checkmark}{c}f(x+h)-\overset{\checkmark}{c}f(x)}{h}$$

$$=\lim_{h\to0}\frac{c[f(x+h)-f(x)]}{h}$$

$$=c\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$

$$=c\,\frac{d}{dx}[f(x)]\checkmark$$

Jul 2-8:26 AM

Prove $\frac{d}{dx}\left[f(x)g(x)\right] = \frac{d}{dx}\left[f(x)\right]g(x) + f(x)\frac{d}{dx}\left[g(x)\right]$

$$\frac{d}{dx}\left[f(x)g(x)\right] = \lim_{h\to 0}\frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0}\frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0}\frac{g(x+h)\left[f(x+h) - f(x)\right] + f(x)\left[g(x+h) - g(x)\right]}{h}$$

$$= \lim_{h\to 0}\left[\frac{g(x+h)\left[f(x+h) - f(x)\right]}{h} + \frac{f(x)\left[g(x+h) - g(x)\right]}{h}\right]$$

$$= \lim_{h\to 0}\frac{g(x+h)\left[f(x+h) - f(x)\right]}{h} + \lim_{h\to 0}\frac{f(x)\left[g(x+h) - g(x)\right]}{h}$$

$$= \boxed{\lim_{h\to 0}\frac{f(x+h) - f(x)}{h}}\boxed{\lim_{h\to 0} g(x+h)} + \boxed{\lim_{h\to 0} f(x)}\boxed{\lim_{h\to 0}\frac{g(x+h) - g(x)}{h}}$$

$$= \frac{d}{dx}\left[f(x)\right]g(x) + f(x)\frac{d}{dx}\left[g(x)\right] \checkmark$$

Jul 2-8:31 AM

find $f'(x)$

1) $f(x) = \frac{3}{4}x^8$ $\quad f'(x) = \frac{3}{4}\cdot 8x^7 = \boxed{6x^7}$

2) $f(x) = \frac{12}{x^3} = 12x^{-3}$ $\quad f'(x) = 12\cdot(-3)x^{-4} = \boxed{\frac{-36}{x^4}}$

3) $f(x) = \pi^2$ $\quad \boxed{f'(x) = 0}$

4) $f(x) = x^{5/3} - x^{2/3}$ $\quad f'(x) = \frac{5}{3}x^{\frac{5}{3}-1} - \frac{2}{3}x^{\frac{2}{3}-1}$

$$= \frac{5}{3}x^{\frac{2}{3}} - \frac{2}{3}x^{-1/3}$$

$$= \frac{5}{3}x^{\frac{2}{3}} - \frac{2}{3x^{1/3}}$$

5) $f(x) = \sqrt{x}\,(x-1)$

$$f'(x) = \frac{1}{2\sqrt{x}}(x-1) + \sqrt{x}\cdot 1$$

$$= \frac{\overset{\sqrt{x}}{\cancel{x}}}{2\cancel{\sqrt{x}}} - \frac{1}{2\sqrt{x}} + \sqrt{x} = \frac{\sqrt{x}}{2} - \frac{1}{2\sqrt{x}} + \sqrt{x}$$

$$= \boxed{\frac{3\sqrt{x}}{2} - \frac{1}{2\sqrt{x}}}$$

Jul 2-8:45 AM

6) $f(x) = (2x^3 + 4x)(3x^2 - 5)$

$f'(x) = (6x^2+4)(3x^2-5) + (2x^3+4x)6x$

$= \underline{\underline{18x^4}} - \underline{30x^2} + \underline{12x^2} - 20 + \underline{\underline{12x^4}} + \underline{24x^2}$

$\boxed{= 30x^4 + 6x^2 - 20}$

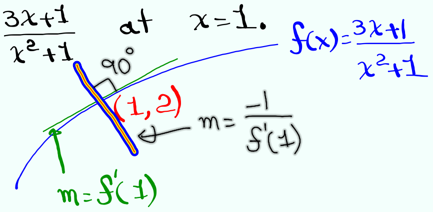
7) $f(x) = \dfrac{x-4}{x+4}$

$$f'(x) = \frac{1(x+4) - (x-4)\cdot 1}{(x+4)^2} = \frac{x+4-x+4}{(x+4)^2} = \boxed{\frac{8}{(x+4)^2}}$$

Jul 2-8:59 AM

Find the eqn of the tan. line and eqn of the normal line to the graph

$f(x) = \dfrac{3x+1}{x^2+1}$ at $x = 1$.



$$f'(x) = \frac{3(x^2+1) - (3x+1)\cdot 2x}{(x^2+1)^2} = \frac{3x^2+3-6x^2-2x}{(x^2+1)^2} = \boxed{\frac{-3x^2-2x+3}{(x^2+1)^2}}$$

$m = f'(1) = \dfrac{-3-2+3}{(1^2+1)^2} = \dfrac{-2}{4} = \dfrac{-1}{2}$

Tan. line

$y - 2 = \frac{-1}{2}(x-1) \Rightarrow \boxed{y = \frac{-1}{2}x + \frac{5}{2}}$

$m = \dfrac{-1}{f'(1)} = \dfrac{-1}{\frac{-1}{2}} = 2$

Normal line

$y - 2 = 2(x-1) \Rightarrow \boxed{y = 2x}$

Jul 2-9:09 AM

Show $\frac{d}{dx}[\csc x] = -\csc x \cot x$

$$\frac{d}{dx}[\csc x] = \frac{d}{dx}\left[\frac{1}{\sin x}\right]$$

$$= \frac{\overset{0}{\cancel{\frac{d}{dx}[1]}}\sin x - 1\cdot \overset{\cos x}{\cancel{\frac{d}{dx}[\sin x]}}}{(\sin x)^2}$$

$$= \frac{-\cos x}{\sin^2 x} = -\frac{\cos x}{\sin x\ \sin x}$$

$$= -\frac{1}{\sin x}\cdot\frac{\cos x}{\sin x} = \boxed{-\csc x \cot x}$$

Jul 2-9:38 AM

Chain Rule

If $y = f(u)$ and $u = g(x)$, then

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

ex: $y = u^3$, $u = 2x+3$ $\Rightarrow y = (2x+3)^3$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = 3u^2\cdot 2$$

$$= 6u^2 = 6(2x+3)^2$$

Jul 2-9:43 AM

$y = \sin u$ $\quad u = x^2+4x-1$ $\quad \Rightarrow y = \sin(x^2+4x-1)$

$$\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx}$$

$$= \cos u\,(2x+4) = \boxed{(2x+4)\cos(x^2+4x-1)}$$

$\frac{d}{dx}\left[2\sqrt{x}+x^2\right]$

$y = \sec u$ $\quad u = 2\sqrt{x}+x^2$

$$\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx} = \sec u \tan u \cdot \left(\frac{1}{\sqrt{x}}+2x\right)$$

$$= \left(\frac{1}{\sqrt{x}}+2x\right)\sec\left(2\sqrt{x}+x^2\right)\tan\left(2\sqrt{x}+x^2\right)$$

Jul 2-9:47 AM

$y = u^4$ $\quad u = \frac{x+1}{x-2}$ $\quad$ Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx}$$

$$= 4u^3 \cdot \frac{1(x-2)-(x+1)\cdot 1}{(x-2)^2} = 4\left(\frac{x+1}{x-2}\right)^3 \cdot \frac{-3}{(x-2)^2}$$

$$= \frac{4(x+1)^3\cdot(-3)}{(x-2)^3(x-2)^2}$$

$$= \frac{-12(x+1)^3}{(x-2)^5}$$

Jul 2-9:53 AM

$y = \sqrt{u}$ $\quad u = \frac{x^2-1}{x^2+1}$ $\quad$ find $\frac{dy}{dx}$

$y = u^{1/2}$

$$\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx}$$

$$= \frac{1}{2\sqrt{u}} \cdot \frac{2x(x^2+1) - (x^2-1)\cdot 2x}{(x^2+1)^2}$$

$$= \frac{1}{2\sqrt{u}} \cdot \frac{\cancel{2x^3} + 2x \checkmark - \cancel{2x^3} + 2x \checkmark}{(x^2+1)^2}$$

$$= \frac{1}{\cancel{2}\sqrt{\frac{x^2-1}{x^2+1}}} \cdot \frac{\overset{2}{\cancel{4}}x}{(x^2+1)^2} = \frac{2x}{\frac{\sqrt{x^2-1}}{\sqrt{x^2+1}}\,(x^2+1)^2}$$

$$= \frac{2x}{\frac{\sqrt{x^2-1}}{(x^2+1)^{1/2}}\,(x^2+1)^2} = \boxed{\frac{2x}{\sqrt{x^2-1}\,(x^2+1)^{3/2}}}$$

Jul 2-10:00 AM

$y = u^3$ $\quad u = \cos t$ $\quad t = x^2 - 10$ $\quad$ find $\frac{dy}{dx}$

$$y = \left(\cos(x^2-10)\right)^3$$

$$\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dt}\,\frac{dt}{dx}$$

$$= 3u^2 \cdot (-\sin t) \cdot 2x$$

$$= -6x\cos^2 t \ \sin t = \boxed{-6x\cos^2(x^2-10)\sin(x^2-10)}$$

Jul 2-10:11 AM

$f(x) = (\underbrace{x^3 - 5x + 10}_{u})^4$ Find $f'(x)$

$f = u^4$ $u = x^3 - 5x + 10$

$f'(x) = 4u^3 \cdot (3x^2 - 5)$

$= 4(x^3 - 5x + 10)^3 (3x^2 - 5)$

$f(x) = \sin x^3$ Find $f'(x)$.

$f(x) = \sin u$, $u = x^3$

$f'(x) = \cos u \cdot 3x^2$ $\boxed{f'(x) = 3x^2 \cos x^3}$

Jul 2-10:17 AM

$f(x) = \tan(\sqrt{x} + 1)$ Find $f'(x)$

$f(x) = \tan u$ $u = \sqrt{x} + 1$

$f'(x) = \sec^2 u \cdot \frac{1}{2\sqrt{x}}$ $\boxed{f'(x) = \frac{1}{2\sqrt{x}} \sec^2(\sqrt{x} + 1)}$

$f(x) = \overset{\checkmark}{\sin}(\overset{\checkmark}{\cos}\ \overset{\checkmark}{x^2})$

$f'(x) = \cos(\cos x^2) \cdot (-\sin x^2) \cdot 2x$

$= \boxed{-2x \cos(\cos x^2) \sin x^2}$

Jul 2-10:23 AM

$$f(x) = (\tan x - x^4)^{3} ✓$$

$$f'(x) = 3(\tan x - x^4)^2 \cdot (\sec^2 x - 4x^3)$$

$$f(x) = (x^2 - 4x)^{100}$$

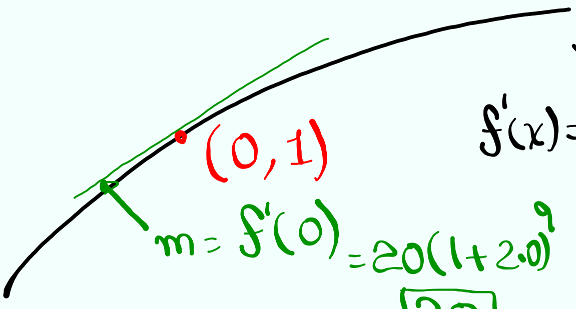
Find $f'(x)$

$$f'(x) = 100(x^2 - 4x)^{99}(2x - 4)$$

$$= 200(x-2)(x^2-4x)^{99}$$

Jul 2-10:33 AM

Find eqn of the tan. line to the graph of $f(x) = (1+2x)^{10}$ at $x = 0$.



$$f(x) = (1+2x)^{10}$$

$$f'(x) = 10(1+2x)^9 \cdot 2$$

$$= 20(1+2x)^9$$

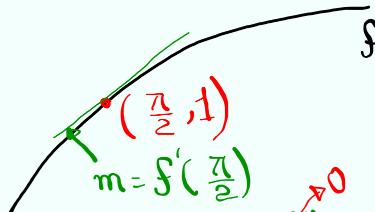
$$m = f'(0) = 20(1+2\cdot 0)^9 = \boxed{20}$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 20(x - 0) \rightarrow \boxed{y = 20x + 1}$$

Jul 2-10:39 AM

Find eqn of the tan. line to the graph of $f(x) = \cos(\cos x)$ at $x = \frac{\pi}{2}$.



$f(\frac{\pi}{2}) = \cos(\cos\frac{\pi}{2})$

$= \cos 0$

$= 1$

$= \sin(\overset{0}{\cos\frac{\pi}{2}})\cdot \overset{1}{\sin\frac{\pi}{2}}$

$= \overset{0}{\sin 0} \cdot 1 = \boxed{0}$

$f(x) = \cos(\cos x)$

$f'(x) = -\sin(\cos x)\cdot -\sin x \cdot 1 = \sin(\cos x)\cdot \sin x$

$y - y_1 = m(x - x_1)$

$y - 1 = 0(x - \frac{\pi}{2})$

$y - 1 = 0 \longrightarrow \boxed{y = 1}$

Jul 2-10:45 AM

Estimate $\sin 91°$

Use your calc

$\sin 91° \approx .9998$

$\sin 91° \approx \sin 90°$

$f(x) = \sin x$

$a = 90° = \frac{\pi}{2}$

$\sin 90° = 1$

$f'(x) = \cos x$

$f'(\frac{\pi}{2}) = \cos\frac{\pi}{2} = 0$

Linear Approx.

$$f(x) \approx f(a) + f'(a)(x - a)$$

$\sin x \approx 1 + 0(x - \frac{\pi}{2})$

$\sin x \approx 1$

$\sin 91° \approx 1$

Jul 2-10:52 AM

Estimate $\tan 46^\circ$

From calc

$\tan 46^\circ \approx \boxed{1.035}5$

$f(x) = \tan x$

$a = 45^\circ = \frac{\pi}{4}$

$f(45^\circ) = f(\frac{\pi}{4}) = \tan\frac{\pi}{4} = 1$

$f'(x) = \sec^2 x$

$f'(45^\circ) = \sec^2 45^\circ$

$= 2$

$180^\circ = \pi$ Rad.

$1^\circ = \frac{\pi}{180}$ Rad.

Linear Approx.

$$f(x) \approx f(a) + f'(a)(x-a)$$

$$\tan x \approx 1 + 2\left(x - \frac{\pi}{4}\right)$$

$$\tan 46^\circ \approx 1 + 2(46^\circ - 45^\circ)$$

$$\approx 1 + 2\cdot 1^\circ$$

$$\approx 1 + 2\cdot\frac{\pi}{180}$$

$$= 1 + \frac{\pi}{90} \approx \boxed{1.0349}$$

$$\approx 1.035$$